

A Novel Investigation Towards the Modeling, Analysis, And MATLAB-Based Simulation of Nonlinear Dynamical Systems for Complex Real-World Applications

Bontha Narendranath Reddy *

Research Scholar, Ph.D. in Mathematics, Bir Tikendrajit University, Cachipur, Imphal, Manipur, India.

Dr Asit Kumar Sen **

Supervisor, Department of Mathematics, Bir Tikendrajit University, Cachipur, Imphal, Manipur, India.

** Corresponding Author Email id: narrendhra.trp@gmail.com*

ABSTRACT

The present thesis provides a comprehensive investigation of nonlinear dynamical systems and their applications across engineering, biomedical sciences, environmental systems, economics, and physical sciences. The study aimed to develop, analyze, and simulate advanced mathematical models capable of capturing complex real-world behaviors that cannot be adequately described by linear approaches. Using MATLAB and Simulink as the primary computational platforms, the research employed nonlinear differential equation models, state-space formulations, and discrete-time systems to analyze stability, bifurcation, limit cycles, chaos, and critical transitions. The findings demonstrated that nonlinear modeling offers significantly deeper insight into system behavior, revealing instability mechanisms, tipping points, and regime shifts observed in practical systems. Applications in power systems, robotics, MEMS/NEMS, epidemic dynamics, cardiac and neural systems, ecology, climate modeling, economic markets, crowd behavior, plasma physics, and nonlinear wave propagation confirmed the universality of nonlinear phenomena. The study further established that nonlinear and robust control strategies are essential for stability and performance under uncertainty. Overall, the thesis highlights nonlinear dynamics and MATLAB-based simulation as indispensable tools for understanding, predicting, and controlling complex real-world systems and supporting informed decision-making.

Keywords: *Nonlinear Dynamical Systems; Advanced Mathematical Modelling; MATLAB/Simulink; Stability and Bifurcation Analysis; Chaos Theory; Nonlinear Control; Numerical Simulation; Real-World Applications.*

1. Introduction

Nonlinear dynamical systems (NDS) are systems in which the evolution of state variables over time is governed by nonlinear relationships, meaning that system responses are not directly proportional to inputs. Unlike linear systems, nonlinear systems exhibit complex behaviors such as bifurcations, limit cycles, chaos, and extreme sensitivity to initial conditions, popularly known as the *butterfly effect*. These characteristics arise due to feedback mechanisms, nonlinear stiffness or damping, parametric excitation, and interaction among system components. Nonlinear dynamical systems are of fundamental importance because most real-world systems are inherently nonlinear, and linear approximations often fail to capture their essential behavior. In engineering applications, nonlinearities significantly affect structural vibrations, rotor dynamics, electrical oscillators, and robotic motion control. Beyond engineering, nonlinear dynamics play a crucial role in biological, ecological, economic, and environmental systems. With advances in computational tools such as MATLAB and Simulink, nonlinear systems can be effectively modeled and simulated, enabling deeper understanding, prediction, and control of complex real-world phenomena.

1.1 Ubiquity in Real-World Systems

Nonlinear dynamical systems are ubiquitous across natural and engineered domains. In mechanical and structural engineering, nonlinear behavior appears in vibrating beams, plates, and rotor-bearing systems due to geometric and material nonlinearities. Electrical and power systems exhibit nonlinear dynamics arising from components such as diodes, transistors, and power electronic converters, leading to harmonics, bifurcations, and instability. In biological and ecological systems, nonlinear interactions govern population dynamics, epidemic spreading, neural activity, and cardiac rhythms. Similarly, economic markets, traffic flow, climate systems, and social networks are dominated by nonlinear feedback, thresholds, and emergent behavior. The widespread presence of nonlinear phenomena highlights the necessity of advanced mathematical modeling and simulation techniques. MATLAB-based computational frameworks provide powerful tools to analyze, visualize, and predict nonlinear behavior across multidisciplinary applications.

1.2 Challenges in Analysis

The analysis of nonlinear dynamical systems presents several fundamental challenges. These systems are typically governed by nonlinear differential or difference equations for which closed-form analytical solutions rarely exist, necessitating approximate methods and numerical simulations. The failure of the superposition principle further complicates system decomposition and analysis. Nonlinear systems often exhibit sensitivity to initial conditions, multiple equilibria, bifurcations, and chaotic behavior, making long-term prediction and stability analysis difficult. Parameter uncertainty, time-varying properties, and external disturbances further increase modeling complexity. High computational demands arise when simulating high-dimensional or multi-physics systems, requiring efficient algorithms and high-performance computing tools. Additionally, designing effective control strategies and validating nonlinear models experimentally remain challenging due to system sensitivity and measurement limitations. Despite these challenges, advanced mathematical

techniques combined with MATLAB-based simulations provide essential frameworks for understanding, analyzing, and controlling nonlinear dynamical systems in real-world applications.

1.3 Types of Advanced Mathematical Approaches

Advanced mathematical approaches provide the theoretical foundation for modeling, analyzing, and simulating nonlinear dynamical systems.

Differential Equations form the primary modeling framework. **Ordinary Differential Equations (ODEs)** describe time-dependent nonlinear systems and capture behaviors such as oscillations, bifurcations, and chaos, commonly analyzed using numerical methods like Runge–Kutta schemes. **Partial Differential Equations (PDEs)** extend this framework to systems dependent on both space and time, such as fluid flow, heat transfer, and wave propagation, where numerical discretization techniques are essential.

Difference equations and discrete-time models represent systems evolving at discrete time intervals, widely used in population studies, economics, and digital control. These models reveal unique nonlinear behaviors such as period-doubling and chaos.

Integral and integro-differential equations incorporate memory and delay effects, enabling realistic modeling of viscoelastic, biological, and feedback-controlled systems.

Hamiltonian and Lagrangian mechanics provide energy-based formulations for nonlinear mechanical systems, facilitating stability analysis and conservation law identification.

Perturbation and approximation methods allow analytical insight into weakly nonlinear systems, supporting the study of stability, resonance, and transitions to chaos.

Nonlinear algebraic methods, including Lyapunov stability theory and bifurcation analysis, are crucial for equilibrium assessment and qualitative behavior prediction.

Numerical and computational techniques enable the simulation of high-dimensional nonlinear systems using methods such as finite difference, finite element, and spectral techniques.

Chaos theory addresses irregular yet deterministic behaviors using tools like Lyapunov exponents and phase-space analysis.

Stochastic and probabilistic methods account for randomness and uncertainty, improving realism in modeling noisy real-world systems.

Control and optimization methods integrate nonlinear modeling with advanced control strategies to achieve stability, robustness, and optimal performance.

1.4 Role of MATLAB in Nonlinear System Analysis

MATLAB plays a vital role in bridging theoretical nonlinear dynamics with practical simulation and analysis. Its built-in solvers efficiently handle nonlinear ordinary, partial, and integro-differential equations, while Simulink provides a flexible block-diagram environment for multi-domain system modeling. MATLAB supports advanced visualization techniques such as phase portraits, bifurcation diagrams, and Poincaré sections, enabling clear interpretation of nonlinear behaviors including chaos

and stability transitions. Parameter sweeps and sensitivity analysis tools facilitate systematic exploration of system dynamics under varying conditions. Through integration with control, optimization, and signal-processing toolboxes, MATLAB enables the design and testing of advanced nonlinear control strategies. By allowing experimentation in a virtual environment, MATLAB reduces cost, risk, and development time, making it an indispensable platform for research and real-world applications in nonlinear dynamical systems.

1.5 Scope and Limitations of the Research

Scope of the Research: This research focuses on advanced mathematical modeling and MATLAB-based simulation of nonlinear dynamical systems relevant to real-world applications. The study involves the formulation of nonlinear models using ordinary differential equations and state-space representations to describe complex system dynamics. Qualitative and numerical analysis techniques such as equilibrium analysis, phase-plane visualization, Lyapunov stability theory, and bifurcation analysis—are employed to investigate system behavior. MATLAB serves as the primary computational tool for numerical solution, simulation, and visualization. The scope extends to selected engineering and applied science applications, demonstrating how MATLAB-based simulations effectively analyze nonlinear system performance under practical operating conditions.

Limitations of the Research: Despite its comprehensive framework, the research has certain limitations. The analysis is restricted to deterministic nonlinear systems and does not explicitly address stochastic dynamics, significant uncertainty, or real-time control implementation. Simulation accuracy depends on modeling assumptions and numerical solvers used in MATLAB, which may introduce approximation errors. Additionally, large-scale hardware validation and real-time experimental implementation are beyond the scope of this study. Computational constraints may also limit the analysis of very high-dimensional or highly complex nonlinear systems. Consequently, while the findings provide valuable insights, their direct applicability to systems with different assumptions or configurations may be limited.

2. Background

Bajpai and Sameer (2025) systematically reviewed nonlinear dynamical systems in decision-making and examined how uncertainty influenced system behavior. They highlighted the role of feedback, instability, and nonlinear interactions in shaping decisions and demonstrated that nonlinear models provided more realistic insights than linear frameworks in complex decision environments.

Fananás-Anaya et al. (2025) investigated the simulation of dynamical systems using attention mechanisms and recurrent neural networks. They demonstrated that neural architectures improved prediction accuracy and captured long-term dependencies in nonlinear systems more effectively than classical numerical methods, especially in data-driven and adaptive modeling scenarios.

de Jong et al. (2025) analyzed uncertainty in limit-cycle oscillations of nonlinear dynamical systems using Fourier generalized polynomial chaos expansion. Their study showed that the method efficiently quantified parameter uncertainty and improved the robustness of nonlinear vibration analysis, particularly for systems exhibiting periodic and oscillatory behavior.

Dong et al. (2024) revisited predictability in dynamical systems by proposing a local data-driven approach. They demonstrated that combining nonlinear dynamics with localized data analysis enhanced short-term prediction accuracy and provided improved insight into system sensitivity, especially in chaotic and high-dimensional systems.

Singh et al. (2024) developed a prescribed-time optimal control framework for nonlinear dynamical systems and validated it using a coupled tank system. Their results showed improved convergence speed, robustness, and stability, highlighting the effectiveness of nonlinear optimal control in practical engineering applications.

Viknesh et al. (2024) introduced the ADAM-SINDy framework for identifying nonlinear dynamical systems. They demonstrated that the proposed optimization approach improved parameter estimation accuracy and computational efficiency, enabling reliable identification of nonlinear models from noisy and limited datasets.

Zhang et al. (2023) proposed a parameter identification framework for nonlinear dynamical systems with Markovian switching. Their study showed that the method accurately captured abrupt regime changes and enhanced modeling precision for systems experiencing random transitions between multiple dynamic modes.

Gudetti et al. (2023) presented a data-driven modeling approach for linear and nonlinear dynamic systems in noise and vibration applications. They demonstrated that machine-learning-based techniques effectively captured system dynamics and improved prediction accuracy compared to traditional physics-based modeling methods.

Haluszczynski et al. (2023) explored machine-learning-based control of dynamical systems using reservoir computing. Their findings showed that next-generation learning models outperformed classical approaches in steering systems toward complex target states, highlighting the potential of learning-based nonlinear control strategies.

Linka et al. (2022) developed Bayesian physics-informed neural networks for modeling real-world nonlinear dynamical systems. They demonstrated that integrating physical laws with probabilistic learning improved prediction reliability, uncertainty quantification, and robustness in systems governed by incomplete or noisy data.

Eshkevari et al. (2022) investigated input estimation in nonlinear systems using probabilistic neural networks. Their results showed enhanced estimation accuracy under noisy conditions, demonstrating the effectiveness of probabilistic learning techniques for system identification and monitoring in nonlinear mechanical systems.

Wabersich and Zeilinger (2021) proposed a predictive safety filter for learning-based control of constrained nonlinear systems. They demonstrated that the framework ensured constraint satisfaction while maintaining performance, enabling safe integration of learning algorithms into real-time nonlinear control systems.

Haluszczynski and Räth (2021) demonstrated the control of nonlinear dynamical systems into arbitrary target states using machine learning. Their study showed that learning-based controllers achieved precise state regulation and adaptability, outperforming conventional control techniques in highly nonlinear environments.

Li et al. (2021) introduced a novel embedding method for characterizing low-dimensional nonlinear dynamical systems. Their approach improved phase-space reconstruction and enhanced the detection of nonlinear features, enabling better system classification and dynamic behavior interpretation.

Wang and Yu (2021) surveyed physics-guided deep learning approaches for dynamical systems. They emphasized how combining physical constraints with deep learning improved interpretability, generalization, and stability, particularly for complex nonlinear systems with limited or noisy data.

Vortmeyer-Kley et al. (2021) proposed a trajectory-based loss function to identify missing terms in bifurcating dynamical systems. Their method successfully recovered hidden nonlinear dynamics and improved model accuracy near bifurcation points and transition regimes.

McDonald and Álvarez (2021) presented a compositional modeling framework for nonlinear dynamical systems using ODE-based random features. Their approach demonstrated improved scalability and expressiveness, enabling efficient learning and simulation of complex nonlinear dynamics.

3. Research Methodology

This chapter outlines a systematic and rigorous methodology adopted to study nonlinear dynamical behavior and control phenomena in engineering and interdisciplinary applications. The methodology emphasizes mathematical modeling, numerical simulation, and analytical interpretation to ensure reproducibility, consistency, and scientific validity.

3.1 Research Design

The study adopted a **quantitative, model-based, and simulation-driven research design** to investigate complex systems governed by nonlinear dynamics. Rather than relying on field data or empirical observation, the research focused on analytically formulated models and computational experimentation. This approach was selected because nonlinear systems exhibit feedback, sensitivity to initial conditions, and emergent behaviors that are inadequately captured by linear or purely empirical methods. The research was exploratory in nature, aiming to analyze nonlinear phenomena such as instability, oscillations, bifurcations, and chaos using differential equations, state-space models, and nonlinear system theory. System parameters and initial conditions were systematically varied to examine regime shifts and tipping points. Analytical techniques including equilibrium analysis, linearization, Jacobian evaluation, eigenvalue analysis, and phase-plane methods were employed to establish stability properties. MATLAB/Simulink served as the primary computational platform for simulation, visualization, and validation through time-domain responses, phase trajectories, bifurcation diagrams, and sensitivity analysis. The integration of analytical rigor with computational modeling ensured a comprehensive and reliable investigation of nonlinear system dynamics.

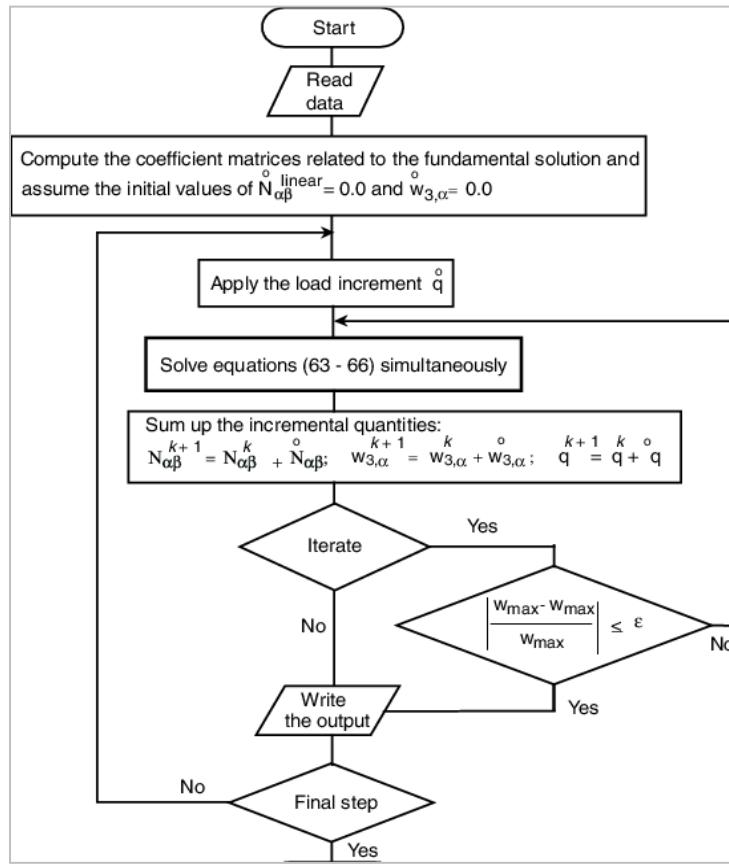


Fig 1: Flowchart of the Iterative Computational Procedure

The research methodology adopted in this study follows a structured and systematic approach to investigate nonlinear dynamical systems using advanced mathematical modelling and MATLAB-based simulation. The study starts from an in-depth literature review where nonlinear dynamics is considered to build a theoretical framework of suitable mathematical models for real-world systems. According to this review, representative nonlinear systems are identified and described by the differential equations and state-space representations with appropriate assumptions and system parameters.

4. Engineering Applications

Power System Stability Using Nonlinear Dynamical Modeling and MATLAB Simulation Observed Result.

- Sudden increase in rotor angle
- Possible loss of synchronism
- Critical clearing time can be estimated

Nonlinear Control of Robots: Robotic systems are inherently nonlinear due to complex kinematics, actuator saturation and joint friction and changing payloads. The use of the correct state-space modelling allows these nonlinear dynamics to be captured and becomes the foundation for advanced control design. Techniques like the feedback linearization are exploited to cancel out non-linear

terms and present desired linear dynamics which facilitate accurate motion control. Sliding-mode control achieves robustness against parameter variations and external disturbances by constraining system trajectories on stable manifolds. Performance improvement is achieved by adaptive control, which modifies controller parameters continuously to account for unknown or variable system dynamics. It can be implemented with a Simulink structure in trajectory tracking and disturbance rejection successfully.

MEMS/NEMS Devices: Micro-Electro-Mechanical Systems (MEMS) and Nano-Electro-Mechanical Systems (NEMS) exhibit strongly nonlinear behavior at micro- and nano-scales, where electrostatic forces, surface effects, and geometric nonlinearities dominate system dynamics. These nonlinearities have large impacts on the performance, stability and reliability of devices. Simplified mathematical models' representations of high-dimensional distributed-parameter systems are frequently used to capture the important nonlinear dynamics. Numerical tools built on MATLAB such as continuation and bifurcation can be used to perform steady-state and dynamic analysis systematically. These simulations allow us to determine resonance frequency shifts, pull-in instability and multicable operating regimes. One of the primary challenges in MEMS/NEMS implicitizations is to comprehend these nonlinear phenomena for efficient design and robust operation of their sensors and actuators.

5. Solitons, Dispersion, and Stability Analysis Using MATLAB

a) Background

Nonlinear waves occur when **nonlinearity and dispersion balance each other**, giving rise to stable, localized structures called **solitons**. Such waves are fundamental in:

- Fluid dynamics (shallow water waves)
- Nonlinear optics (fiber-optic communication)
- Solid-state physics and materials
- Plasma and Bose–Einstein condensates

Linear wave theory fails to explain **shape-preserving propagation, wave interaction, and stability**, necessitating nonlinear PDE-based modeling.

b) Mathematical Origin of Nonlinear Waves

A general nonlinear dispersive wave equation can be written as:

$$\frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} + \alpha u \frac{\partial u}{\partial x} + \beta \frac{\partial^3 u}{\partial x^3} = 0$$

were

- $u(x, t)$: wave amplitude
- α : nonlinearity coefficient
- β : dispersion coefficient

The **competition between the nonlinear term uu_x and dispersive term u_{xxx}** enables soliton formation.

c) Korteweg–de Vries (KdV) Equation (Fluid Waves)
KdV Equation

$$\frac{\partial u}{\partial t} + 6u \frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0$$

Exact Soliton Solution

$$u(x, t) = A \operatorname{sech}^2 \left[\sqrt{\frac{A}{2}} (x - ct) \right], c = A$$

- Shape-preserving
- Elastic collision between solitons
- Infinite conserved quantities

d) Nonlinear Schrödinger Equation (Optics & Quantum Media)
NLSE

$$i \frac{\partial \psi}{\partial t} + \frac{1}{2} \frac{\partial^2 \psi}{\partial x^2} + \gamma |\psi|^2 \psi = 0$$

where

- $\psi(x, t)$: complex wave envelope
- γ : Kerr nonlinearity

Fundamental Soliton

$$\psi(x, t) = \eta \operatorname{sech}(\eta x) e^{i\eta^2 t/2}$$

Applications:

- Optical fiber communication
- Bose–Einstein condensates
- Plasma envelope waves

e) Stability and Modulational Instability

Perturb a plane wave:

$$\psi = (\psi_0 + \epsilon) e^{i\theta}$$

Leads to dispersion relation:

$$\omega^2 = k^2(k^2 - 2\gamma |\psi_0|^2)$$

Instability occurs when:

$$k^2 < 2\gamma |\psi_0|^2$$

- Explains rogue waves
- Predicts soliton breakup

f) Soliton Interaction and Integrability

Soliton collisions satisfy:

- Phase shifts without amplitude loss
- Conservation of energy, momentum, and mass

Mathematically:

$$\int u \, dx = \text{const}, \int u^2 \, dx = \text{const}$$

These invariants guarantee long-term stability.

g) Numerical Modeling in MATLAB

Finite-Difference Method (FDM)

Spatial discretization:

$$\frac{\partial u}{\partial x} \approx \frac{u_{i+1} - u_{i-1}}{2\Delta x}$$

Time integration:

- Explicit Runge–Kutta
- Implicit Crank–Nicolson (for stiffness)

Spectral (Fourier) Method

Represent solution as:

$$u(x, t) = \sum_k \hat{u}_k(t) e^{ikx}$$

- High accuracy
- Ideal for periodic domains
- Efficient for soliton dynamics

Time stepping:

- Split-step Fourier method
- Exponential time differencing (ETDRK4)

h) Bifurcation and Transition to Turbulence

As nonlinearity increases:

Linear waves → Solitons → Breathers → Wave turbulence

This corresponds to

- Hopf bifurcation
- Modulational instability
- Chaotic wave fields

This section demonstrated that nonlinear dynamical modeling combined with MATLAB-based simulation is a powerful and unifying approach for analyzing complex real-world systems. In engineering applications, it facilitated the precise stability evaluation of power systems, robust control design for robot manipulators, and dependable design knowledge of micro electro mechanical machines (MEMs) / Nano electro mechanical machines (NEM's). In bio-medical systems, nonlinear models adequately described epidemic propagation, cardiac rhythms, and neural oscillation, therefore contributing support to the design of control and intervention policies. Environmental, economic, and social systems displayed bifurcations, tipping points, and chaotic behaviour under stress whereas physical systems exhibited chaos, turbulence and soliton dynamics. In general, the nonlinear analysis was much more insightful, realistic, and predictive in comparison to conventional linear methods.

6. Conclusion and Future Work

The research confirmed that nonlinearity is a fundamental property of real-world systems, not a secondary effect. MATLAB/Simulink proved to be a powerful and flexible platform for modeling, simulation, stability analysis, and control validation of nonlinear systems. Bifurcation and stability analysis emerged as critical tools for identifying early-warning thresholds in engineering, biomedical, environmental, and socio-economic systems. The study further established that effective control strategies must be nonlinear and robust, as linear control methods are inadequate under uncertainty and strong nonlinear interactions. Overall, nonlinear modeling and simulation provided actionable insights for real-world decision-making and risk mitigation. Future research should incorporate higher-fidelity models using real datasets and differential-algebraic formulations for stronger validation. Hybrid approaches combining nonlinear models with machine learning can improve prediction under uncertainty. Real-time hardware-in-the-loop implementation of nonlinear controllers is recommended to bridge theory and practice. Further extensions may include multi-scale and multi-physics coupling and advanced uncertainty quantification techniques to enhance robustness and reliability.

References

1. Bajpai, S., & Sameer, A. (2025). The dynamics of uncertainty: a systematic review of non-linear dynamical systems in decision-making. *Nonlinear Dynamics*, 1-17.
2. Fananás-Anaya, J., López-Nicolás, G., & Sagüés, C. (2025). Dynamical system simulation with attention and recurrent neural networks. *Neural Computing and Applications*, 37(4), 2711-2731.
3. de Jong, L., Clasen, P., Mueller, M., & Römer, U. (2025). Uncertainty analysis of limit cycle oscillations in nonlinear dynamical systems with the Fourier generalized Polynomial Chaos expansion. *Journal of Sound and Vibration*, 607, 119017.
4. Dong, C., Faranda, D., Gualandi, A., Lucarini, V., & Mengaldo, G. (2024). Revisiting the predictability of dynamical systems: a new local data-driven approach. *arXiv preprint arXiv:2409.14865*.
5. Singh, V. K., Kamal, S., Bandyopadhyay, B., Ghosh, S., & Dinh, T. N. (2024). Prescribed-time optimal control of nonlinear dynamical systems with application to a coupled tank system. *IEEE Transactions on Automation Science and Engineering*.

6. Viknesh, S., Tatari, Y., Christenson, C., & Arzani, A. (2024). Adam-sindy: An efficient optimization framework for parameterized nonlinear dynamical system identification. *arXiv preprint arXiv:2410.16528*.
7. Zhang, Z., Shen, Q., & Wang, X. (2023). Parameter identification framework of nonlinear dynamical systems with Markovian switching. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 33(12).
8. Gudetti, J. P., Yazdi, S. J. M., Baqersad, J., Peters, D., & Ghamari, M. (2023). *Data-Driven Modeling of Linear and Nonlinear Dynamic Systems for Noise and Vibration Applications* (No. 2023-01-1078). SAE Technical Paper.
9. Haluszczynski, A., Köglmayr, D., & Räth, C. (2023, June). Controlling dynamical systems to complex target states using machine learning: next-generation vs. classical reservoir computing. In *2023 International Joint Conference on Neural Networks (IJCNN)* (pp. 1-7). IEEE.
10. Linka, K., Schäfer, A., Meng, X., Zou, Z., Karniadakis, G. E., & Kuhl, E. (2022). Bayesian physics informed neural networks for real-world nonlinear dynamical systems. *Computer Methods in Applied Mechanics and Engineering*, 402, 115346.
11. Eshkevari, S. S., Cronin, L., Eshkevari, S. S., & Pakzad, S. N. (2022). Input estimation of nonlinear systems using probabilistic neural network. *Mechanical Systems and Signal Processing*, 166, 108368.
12. Wabersich, K. P., & Zeilinger, M. N. (2021). A predictive safety filter for learning-based control of constrained nonlinear dynamical systems. *Automatica*, 129, 109597.
13. Haluszczynski, A., & Räth, C. (2021). Controlling nonlinear dynamical systems into arbitrary states using machine learning. *Scientific reports*, 11(1), 12991.
14. Li, D., Cao, M., Manoach, E., & Ragulskis, M. (2021). A novel embedding method for characterization of low-dimensional nonlinear dynamical systems. *Nonlinear Dynamics*, 104(1), 125-148.
15. Wang, R., & Yu, R. (2021). Physics-guided deep learning for dynamical systems: A survey. *ACM Computing Surveys*.
16. Vortmeyer-Kley, R., Nieters, P., & Pipa, G. (2021). A trajectory-based loss function to learn missing terms in bifurcating dynamical systems. *Scientific Reports*, 11(1), 20394.
17. McDonald, T., & Álvarez, M. (2021). Compositional modeling of nonlinear dynamical systems with ODE-based random features. *Advances in neural information processing systems*, 34, 13809-13819.